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SE-6001

B. E. I (Sem. I) (All) Examination

April / May – 2011

Engineering Mathematics-I

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. 1 (Sem. 1) (All)**

Name of the Subject :

☛ **Engineering Mathematics-1**

☛Subject Code No. : **6 0 0 1** ☛Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

(4) Draw the figures whenever it is necessary.

1 (a) Solve the following : **10**

(1) If $y = \cos^3 x$ then find y_n.

(2) Define Maclarin's series for the function of one variable and expand e^x using it.

(3) Give the polar form of complex number $\frac{1+i}{1-i}$

(4) Find n^{th} derivative of $e^{ax} \cos(bx + c)$.

(5) Define radius of curvature. State its formula in cartesian coordinates.

(b) Attempt the following : **5**

(1) State and prove Leibneitz's theorem to find n^{th} derivative of product of two functions. **04**

(2) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that **03**

$$x^2 y_{n+2} + (2n+1)\, x y_{n+1} + 2n^2 y_n = 0.$$

(3) For $y = \frac{x^n}{1+x}$ obtain its n^{th} derivative y_n. **03**

2 (a) Derive the radius of curvature formula for the polar curver = f(θ). **03**

(b) Attempt any two of the following. **06**

(1) Expand f(x) - $\log_e x$ using Taylor's series expansion and hence obtain the value of $\log_e 1.1$

(c) Attempt any two of the following. **06**

(1) Evaluate $\lim_{x\to 0}\left(\frac{\cot x}{\cot 2x}\right)$.

(2) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$.

(3) Evaluate $\lim_{x\to 0}\left(\sqrt{x+1}-\sqrt{x}\right)\log\left(\frac{1}{x}\right)$.

3 (a) State and prove DeMover's theorem **04**

(b) Attempt any two of the following. **08**

(1) Prove that $\tan^{-1} x=\frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$

(2) If $\alpha+\beta=tanh\left(x+i\frac{\pi}{4}\right)$ then prove that $\alpha^2+\beta^2=1$

(3) Prove that $\left(1+\sqrt{3}i\right)^n+\left(1-\sqrt{3}i\right)^n=2^{n+1}\cos\left(\frac{n\pi}{3}\right)$

(c) Attempt any one of the following : **03**

(1) Find modulus and argument of i^{1-r}

(2) Prove that $\log\left(\frac{1}{1-e^{i\theta}}\right)=\log\left[\frac{1}{2}\cos ec\left(\frac{\theta}{2}\right)\right]+i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)$

4 (a) Do as directed. **10**

(1) Define exact differential equation and give its solution.

(2) Give arc length formula for the parametric curve x = f(t) and y = g(t).

(3) State the first order, first degree linear differential equation and write its general solution.

(4) Define order and degree of the differential equation.

(5) Sketch the curve r = a cos 2θ.

(b) Attempt the following. **06**

(1) Trace $9a\,zy^2 = x(x-3a)^2$

(2) Trace the curve $r = a(1+\cos\theta)$

(c) Find the surface area of the solid formed by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line. **03**

5 (a) Solve any **three** of the following : **09**

(1) $(x+y)^2 \frac{dy}{dx} = a^2$

(2) $\frac{dy}{dx} = e^{x-y} + e^{2l0gx-y}$

(3) $xdy - ydx = \sqrt{x^2+y^2}\,dx$

(4) $\left(x^2+1\right)\frac{dy}{dx} + 2xy = 4x^2$

(b) Attempt any **two** of the following : **06**

(1) $y = 2x + p^2$

(2) $(x-a)\,p^2 + (x-y)\,p - y = 0$

(3) $x^2p^3 + y(1+x^2y)\,p^2 + y^3p = 0$

6 Attempt any two of the following : **16**

(1) State and formulate SIS - model for spread of epidemic disease. Also obtain its solution with its interpretation.

(2) A resistance of 70 ohms and inductance of 0.80 Henry are connected in series with a battery of 10 volts. Determine the expression for current as a function of time at t=0.

(3) State and formulate differential equation model for electric network (RC-circuit with variable emf). Obtain its solution, analysie it and write its interpretation.
